Axial Torsion Waves in Metric-affine Gravity

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Structure of talk

- Mathematical model
- PP-waves with axial torsion
- New vacuum solutions of quadratic metric-affine gravity
- Physical interpretation

Quadratic metric-affine gravity

Spacetime a connected real 4–manifold M with a Lorentzian metric g and an affine connection Γ,

$$\nabla_\mu u^\lambda = \partial_\mu u^\lambda + \Gamma^\lambda{}_{\mu\nu} u^\nu .$$

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An *independent* linear connection Γ distinguishes MAG from GR - g and Γ viewed as two totally independent quantities. Action is

$$S := \int q(R),$$

where $q(R)$ is a Lorentz invariant purely quadratic form on curvature.

Field equations

Euler–Lagrange equations

$$\partial S/\partial g = 0, \quad (1)$$
$$\partial S/\partial \Gamma = 0. \quad (2)$$

The Yang–Mills action for the affine connection is a special case

$$q(R) := R^{\kappa}{}_{\lambda\mu\nu} R^{\lambda}{}_{\kappa}{}^{\mu\nu}.$$

Classical pp-waves

Definition

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A *pp-wave* is a Riemannian spacetime whose metric can be written locally in the form

$$\mathrm{d}s^2 = 2\,\mathrm{d}x^0\,\mathrm{d}x^3 - (\mathrm{d}x^1)^2 - (\mathrm{d}x^2)^2 + f(x^1, x^2, x^3)\,(\mathrm{d}x^3)^2$$

in some local coordinates.

Well known spacetimes in GR, simple formula for curvature.

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A a real vector field $A = k(\phi)\, l$, where $k : \mathbb{R} \mapsto \mathbb{R}$ arbitrary

$$\varphi : M \to \mathbb{R}, \qquad \varphi(x) := \int_M l \cdot dx\,.$$

Torsion (3) clearly purely axial.

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- Torsion of a generalised pp-wave is

$$T = \mp \frac{i}{2} k \; l \wedge m \wedge \overline{m}.$$

- Ricci curvature is

$$Ric = \frac{1}{2}(f_{11} + f_{22} - k^2) l \otimes l.$$

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- Note that by $\{Ric\}$ we denote the Riemannian Ricci curvature.
- Condition $\{\nabla\}\{Ric\} = 0$ implies that $f_{11} + f_{22} = C$.
- Result also holds if Ric is assumed to be parallel.
- Accepted for publication *Advances in High Energy Physics*.

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Conjecture

Generalised pp-waves with purely axial torsion of parallel $\{Ric\}$ are solutions of the system of equations (1), (2).

Physical interpretation?

- Curvature of generalised pp-waves is split.
- Torsion and torsion generated curvature are waves traveling at the speed of light.
- Underlying pp-space can be viewed as the ‘gravitational imprint’ created by wave of some massless field.
- Mathematical model for some massless particle, similarly to V. Pasic and E. Barakovic, Gen. Relativ. Gravitation **46**, 10 (2014)?

Metric-affine vs Einstein-Weyl

Look at massless Dirac (or Weyl) action

$$S_{\mathrm{W}} := 2i \int \left(\xi^a \, \sigma^\mu{}_{a\dot{b}} \, (\nabla_\mu \bar{\xi}^{\dot{b}}) \; - \; (\nabla_\mu \xi^a) \, \sigma^\mu{}_{a\dot{b}} \, \bar{\xi}^{\dot{b}} \right),$$

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$$S_{\mathrm{EW}} := k \int \mathcal{R} + S_{\mathrm{W}},$$

$$\partial S_{\mathrm{EW}} / \partial g = 0,$$

$$\partial S_{\mathrm{EW}} / \partial \xi = 0.$$

Thank You!